

Ruling out a higher spin field solution to the cosmological constant problem

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Abstract

We consider the modification of Newton's gravity law in Dolgov's higher spin models [1, 2] designed to compensate the cosmological constant. We find that the effective Planck mass is unacceptably large in these models. We also point out that the properties of gravitational waves are entirely different in these models as compared to general relativity.

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1 Introduction

One of the most challenging problems in contemporary physics is the cosmological constant problem: observationally allowed values of the vacuum energy density are many orders of magnitude larger than those natural from particle physics point of view (for a review see, e.g., ref.[3]). An attractive approach towards the solution of this problem would be to introduce a compensating field which interacts with gravity in such a way as to relax dynamically to a state with zero or very small net energy density. Attempts to make use of a scalar field for this purpose have failed so far [4, 5], so it was suggested by Dolgov [1, 2] that vector or tensor fields may do the job. The latter mechanism is based on the observation that some components of higher spin fields may develop an instability if the cosmological constant is large and

the Universe is rapidly expanding. As a result of the instability, these components grow, and their contribution to the energy-momentum tensor slows down the expansion of the Universe thus almost completely compensating the effect of the cosmological constant. A peculiarity of this mechanism is that the Lorentz invariance is strongly broken in the gravitational sector.

In this note we point out that the large fields and strong breaking of Lorentz invariance, inherent in Dolgov's models, have drastic effects on gravitational interactions. In particular, Newton's gravity gets modified in an unacceptable manner. Also, the properties of gravitational waves become entirely different from the conventional ones. Hence, the mechanism proposed in refs.[1, 2] is not acceptable phenomenologically.

2 Compensation mechanism

In this paper we will consider in some detail a variant of the compensation mechanism of ref.[1, 2] which makes use of vector field. The analysis of the models with tensor fields is very similar, and leads to the same conclusions. The action proposed in ref.[2] is

$$S = S_G + S_{vac} + S_A$$

where

$$\begin{aligned} S_G &= -\frac{M^2}{16\pi} \int d^4x \sqrt{-g} R, \\ S_{vac} &= \int d^4x \sqrt{-g} \epsilon_{vac}, \\ S_A &= \frac{\eta}{2} \int d^4x \sqrt{-g} D_\mu A_\nu D^\mu A^\nu. \end{aligned}$$

Here we keep M as a free parameter not necessarily equal to the Planck mass; ϵ_{vac} is the vacuum energy density of ordinary matter (whose effect on the expansion of the Universe is aimed to be compensated), A_μ is the compensating vector field, and η is either $+1$ or -1 , depending on the sign of ϵ_{vac} . The vacuum energy density ϵ_{vac} is expected to be determined by some particle physics scale; say, the contribution of electroweak physics is naturally of order

$$\epsilon_{vac} \sim (100 \text{ GeV})^4.$$

If not for the field A_μ , the Universe would expand way too fast.

A homogeneous spatially flat solution to the field equations with vanishing spatial components A_i has the following large time asymptotics,

$$A_0 = t\sqrt{\epsilon_{vac}/2}, \quad (1)$$

$$H = \frac{1}{t}, \quad (2)$$

where H is the Hubble parameter. The large value of \dot{A}_0 compensates the effect of the vacuum energy density, ϵ_{vac} , so the Universe evolves slowly at large t . This happens because

$$\dot{A}_0^2 \sim A_0^2 H^2 \sim \epsilon_{vac}, \quad (3)$$

which is the major ingredient of the compensation mechanism.

It is clear from eq.(3) that the present value of A_0 is very large,

$$A_0 \sim \frac{\sqrt{\epsilon_{vac}}}{H}.$$

Unless ϵ_{vac} is less than $(10^{-3} \text{ eV})^4$ (i. e., unless no compensation of the cosmological constant is needed), this value at the present epoch is much greater than the Planck mass,

$$A_0 \gg M_{Pl}. \quad (4)$$

As an example, at $\epsilon_{vac} \sim (100 \text{ GeV})^4$ one has $A_0 \sim 10^{46} \text{ GeV}$.

3 Modified gravity law

Let us study perturbations about the background determined by eqs.(1) and (2). We consider time scales and wavelengths of these perturbations much smaller than the expansion time and Hubble radius of the Universe, respectively. It is these perturbations that are relevant to Newton's gravity law and emission of gravitational waves. Under these conditions the largest parameter entering the quadratic part of the perturbed action S_A is the present value of A_0 . The time derivatives of A_0 and background metrics may be

safely ignored; in particular, background metrics is effectively Minkowskian. The quadratic part of S_A then takes the following form,

$$S_A^{(2)} = \frac{\eta}{2} \int \left[\partial_\mu c_\nu \partial^\mu c^\nu - A_0 \partial^\mu c^\nu (\partial_\mu h_{\nu 0} + \partial_\nu h_{\mu 0} - \partial_0 h_{\mu\nu}) \right. \\ \left. + \frac{A_0^2}{4} (\partial_\mu h_{\nu 0} + \partial_\nu h_{\mu 0} - \partial_0 h_{\mu\nu}) (\partial^\mu h^{\nu 0} + \partial^\nu h^{\mu 0} - \partial^0 h^{\mu\nu}) \right] \quad (5)$$

where c_μ and $h_{\mu\nu}$ are perturbations of the vector field and metrics, respectively. The quadratic parts of S_G and S_{vac} are conventional and we do not write them explicitly.

It follows from eq.(5) that c_ν obeys

$$\partial^2 c_\nu = \frac{A_0}{2} \left(\partial^2 h_{\nu 0} + \partial^\mu \partial_\nu h_{\mu 0} - \partial^\mu \partial_0 h_{\mu\nu} \right). \quad (6)$$

Therefore, $c \sim A_0 h$, so the Lagrangian in eq.(5) is of order $A_0^2 (\partial h)^2$.

Let us now consider gravitational field of an external source (ordinary matter). Upon excluding c_μ from the field equations through eq.(6), one obtains an equation for $h_{\mu\nu}$ which has the following structure,

$$M^2 \partial^2 h + \epsilon_{vac} h + A_0^2 \partial^2 h = T^{ext}. \quad (7)$$

where $T_{\mu\nu}^{ext}$ is the external energy-momentum tensor. As we consider relatively large momenta, $\partial^2 h \gg H^2 h$, eq.(3) implies that the second term here is negligible as compared to the third one.

At $M^2 \gg A_0^2$, eq.(7) is the usual weak field limit of the Einstein equations. However, the Newton gravity constant is wrong, because $M \gg M_{Pl}$ (see eq.(4)). Hence, we are lead to consider the opposite case, $A_0 \gg M$. Neglecting the first two terms in eq.(7) we find that it has the following explicit form,

$$T_{\mu\nu}^A + T_{\mu\nu}^{ext} = 0 \quad (8)$$

where

$$T_{\mu\nu}^A = \frac{\eta}{2} \left\{ A_0^2 \left[g_{\mu 0} (-\partial^2 h_{\nu 0} - \partial_\nu \partial^\lambda h_{\lambda 0} + \partial_0 \partial^\lambda h_{\lambda\nu}) \right. \right. \\ \left. \left. + g_{\nu 0} (-\partial^2 h_{\mu 0} - \partial_\mu \partial^\lambda h_{\lambda 0} + \partial_0 \partial^\lambda h_{\lambda\mu}) + \partial_\mu \partial_0 h_{\nu 0} + \partial_\nu \partial_0 h_{\mu 0} - \partial_0^2 h_{\mu\nu} \right] \right. \\ \left. + A_0 \left[g_{\mu 0} (\partial^2 c_\nu + \partial^\lambda \partial_\nu c_\lambda) + g_{\nu 0} (\partial^2 c_\mu + \partial^\lambda \partial_\mu c_\lambda) - \partial_\mu \partial_0 c_\nu - \partial_\nu \partial_0 c_\mu \right] \right\} \quad (9)$$

In the case of static source, the perturbations c_ν and $h_{\mu\nu}$ can be chosen time-independent, and we further impose the Coulomb gauge condition, $\partial_i h_{i0} = 0$. Then one finds from eq.(6) that $c_0 = (A_0/2)h_{00}$, and 00-component of eq.(9) becomes

$$\frac{\eta}{2}A_0^2\partial_i^2 h_{00} + T_{00}^{ext} = 0.$$

This is the Newton's gravity law (or anti-gravity law, depending on the sign of η), but again with the wrong gravitational constant. We conclude that the effective Planck mass is presently too large in the Universe evolving according to eqs.(1) and (2).

4 Modified gravitational waves

While the above argument by itself demonstrates that the mechanism of refs.[1, 2] is not viable, there is yet another undesirable feature of this mechanism which illustrates the danger of strong breaking of Lorentz invariance. Indeed, let us consider gravitational waves in this model. These are conveniently analyzed in the gauge $c_0 = 0$, $h_{0i} = 0$ (the fact that these gauge conditions may indeed be imposed can be seen by inspection of the transformation laws of c_μ and $h_{\mu\nu}$ under general coordinate transformations). In this gauge eq.(6) becomes

$$\begin{aligned}\partial^2 h_{00} &= 0, \\ \partial^2 c_i &= \frac{A_0}{2}(\partial_i \partial_0 h_{00} + \partial_j \partial_0 h_{ij}),\end{aligned}$$

while the sourceless equation (8) reads

$$\begin{aligned}A_0 \partial_0^2 h_{00} - 2\partial_0 \partial_i c_i &= 0, \\ A_0 \partial_0 \partial_j h_{ij} + 2\partial_i \partial_j c_j + \partial_i^2 c_i &= 0, \\ \partial_0(A_0 \partial_0 h_{ij} + \partial_i c_j + \partial_j c_i) &= 0.\end{aligned}$$

This set of equations has only longitudinal and vectorial solutions for metric perturbations, the latter having a peculiar dispersion law:

Longitudinal ($p_0^2 = \mathbf{p}^2$):

$$h_{ij} = p_i p_j d(\mathbf{p}),$$

$$\begin{aligned} h_{00} &= -p_i^2 d(\mathbf{p}), \\ c_i &= -\frac{A_0}{2} p_0 p_i d(\mathbf{p}), \end{aligned}$$

Vectorial ($p_0^2 = \mathbf{p}^2/2$):

$$\begin{aligned} h_{ij} &= p_i d_j(\mathbf{p}) + p_j d_i(\mathbf{p}), \\ h_{00} &= 0, \\ c_i &= -A_0 p_0 d_i(\mathbf{p}), \end{aligned}$$

where $d(\mathbf{p})$ and $d_i(\mathbf{p})$ are arbitrary amplitudes satisfying the only condition $p_i d_i = 0$. These solutions are drastically different from transverse traceless gravitational waves of general relativity, and their emission almost certainly will not pass observational tests.

5 Discussion

The problem with Newton's gravity law which rules out the mechanism of refs.[1, 2] is analogous to the modification of gravity inherent in compensation mechanisms invoking scalar fields [4]. There, too, the effective Planck mass is unacceptably large at the present epoch. In addition to this problem, models with vector and/or tensor compensating fields lead to unconventional properties of gravitational waves; in particular, these do not necessarily travel with the speed of light because of the broken Lorentz invariance.

It appears that undesirable modification of gravity is a generic property of the mechanisms aimed at compensating the cosmological constant: all models proposed so far are ruled out because of this property. It remains to be understood whether there is a way out of this difficulty, and whether realistic compensation models may be constructed.

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